Phase motion: a program for studying longitudinal beam optics in a synchrocyclotron

Wiel Kleeven and Emma Pearson

One of the tasks in the development of a synchrocyclotron concerns the specification of the RF-frequency and voltage tables as a function of time. Two important quantities depend on this: i) the efficiency of capturing injected ions into the RF-bucket and ii) the stability of the RF bucket during the acceleration. A third determining factor is the K-value of the magnetic field (relating the change in revolution frequency to the change of energy). It is possible to study these processes with a full 3-D (transverse and longitudinal) tracking code but such calculations are complex and time-consuming. The purpose of the current study is to develop a simplified tracking code where only the longitudinal motion is calculated. The central region is decoupled from the longitudinal effects and is considered as a ‘filter’ which excludes certain ranges of RF phase which will in practice be lost horizontally or vertically during the first few turns. This is an approximation, but it serves as a helpful and fast additional calculation tool for study of the RF-frequency and voltage curves and it also allows to follow the particle during the full acceleration cycle. The program phase-motion numerically integrates the equations of longitudinal motion (the phase-equations) in a synchrocyclotron. As an example the problem of capture in the S2C2 is studied.

AN EXAMPLE OF APPLICATION

- Ion capture in the S2C2 RF bucket was studied
- RF frequency curve based on actual RF-system design
- Maximum RF frequency can be tuned with $\pm 2$ MHz with vertical stubs placed on the RF line
- Three cases are studied with different maximum RF-frequencies as shown in the table.

### OUTLINE OF THE PROGRAM

- Longitudinal canonical variables are the RF phase $\phi$ and the energy difference $\Delta E$ between particle $E_i$ and the synchronous particle $E_s$. Independent variable is time $t$.
- The properties of the synchronous particle are calculated by a separate program.
- An inner time domain ($t_{in} - t_{in}^{\text{max}}$) is defined for which a synchronous particle exists. This is the case if the RF frequency is smaller than the revolution frequency in the cyclotron center and larger than the revolution frequency at the extraction radius.
- In the outer domain the synchronous phase and energy are put to zero.
- $F_{RF}(t)$ and $V(t)$ are read from an input file.
- Equations of motion are integrated with a 5th order Runge-Kutta with adaptive stepsize.
- Polynomial interpolation is used in the frequency table and the synchronous particle table. Order of polynomial can be chosen by the user.

#### EQUATIONS OF MOTION

- $\frac{d\phi}{dt} = \frac{2\pi F_{RF}(t)}{\lambda} \left(1 - \frac{h}{F_{RF}}\right)$
- $\frac{d\Delta E}{dt} = \frac{F_{RF}}{\lambda} N_{Q} \frac{\partial}{\partial t} \left(\sin \phi - \sin \phi_s\right)$
- $\phi_s$ = synchronous phase
- $F_{RF}$ = RF frequency
- $\lambda$ = revolution frequency
- $P$ = dee voltage
- $h$ = harmonic mode
- $N$ = number of gaps
- $q$ = particle charge

### SYNCHRONOUS PARTICLE

$$\frac{\Delta E}{\Delta E_{\text{max}}} = \frac{\sin \phi}{\sin \phi_s} = \frac{2n \cdot \cos \phi_s}{E_s \cdot \cos \phi_s}$$

### CONCLUSION

- The developed program allows studying ion capture and RF bucket stability for varying magnetic field-maps and RF frequency tables.
- The studied example confirms the trends that were found in full 3D tracking calculations that were made by AIMA.